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Brane-world cosmology, bulk scalars and perturbations

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Abstract: We investigate aspects of cosmology in brane world theories with a bulk scalar field. We concentrate on a recent model motivated from supergravity in singular spaces. After discussing the background evolution of such a brane-world, we present the evolution of the density contrast. We compare our results to those obtained in the (second) Randall-Sundrum scenario and usual 4D scalar-tensor theories.

Keywords: Supergravity Models, Cosmology of Theories beyond the SM, Large Scale Structure Formations, Physics of the Early Universe.

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Contents

1. Introduction

The idea that we live on a hypersurface embedded in a higher dimensional space has sparked a lot of interest recently. These theories are motivated from string theory, where higher-dimensional objects, such as D-branes, play an essential role [1]. Similarily, compactifying M-theory (or its effective low-energy limit: 11-D supergravity) on a $S_1/\mathbb{Z}_2$-orbifold and compactifying six dimensions on a Calabi-Yau manifold, results in a five-dimensional brane world scenario with two hypersurfaces, each located at the orbifold fixed points (see e.g. [2] and [3]).

Brane world theories predict that our universe was higher-dimensional in the past. Because of this, there is the hope that certain questions which cannot be answered within the context of the standard model of cosmology, can be addressed within these theories. Furthermore, cosmology should be a way to severely constrain parameters in these models.

So far, most cosmological considerations of brane worlds centered around the one-brane scenario of Randall and Sundrum [4]. In this model, the three-dimensional brane universe is embedded in a 5-dimensional Anti-de Sitter (AdS) spacetime. In particular, the bulk-space is empty, the only contribution to the curvature comes from the negative cosmological constant in the bulk. This simple model already leads to new effects which are interesting for cosmology [5]. However, most scenarios motivated from particle physics predict matter in the bulk, such as scalar fields. In 5D heterotic M-theory, for example, one particular scalar field measures the deformation of the Calabi-Yau manifold, on which six other small dimensions are compactified [3]. Other models, motivated by supergravity (SUGRA), also predict bulk matter, whose form is dictated by the field theory under consideration.

Cosmology may be a fruitful field where the above ideas can be tested. As such, the study of the evolution of cosmological perturbations is extremely important [6], because the higher-dimensional nature of the world can leave traces in the distribution of matter and/or anisotropies in the microwave sky. A lot of papers investigated different aspects of perturbations in brane world scenarios [7].

The aim of this paper is to develop an understanding of the evolution of perturbations in brane world scenarios, in which scalar field(s) are present in the bulk. As a toy model, we will use a cosmological realization of a supergravity model in singular spaces [8]. The evolution of the brane world was discussed in depth in [9] and [10] (see also [11] for a discussion on brane cosmology and bulk scalar fields). In particular four different cosmological eras have been identified in this model. At high energy above the brane tension the cosmology is non-conventional before entering the radiation epoch where the scalar field is frozen. After matter-radiation equality the scalar field starts evolving in time leading to a slow-down of the expansion rate compared to FRW cosmology in the matter dominated era. Eventually the scalar field dynamics becomes the dominant one leading to a supergravity era. Requiring that coincidence between the matter and scalar field energy densities occurs in the recent past leads a cosmological constant with a fine-tuning of the supersymmetry breaking tension on the brane. In the supergravity era it has been shown that the observed acceleration of the expansion of the universe can be understood, i.e. the computed acceleration paremeter $q_0 = -4/7$ is within the experimental ball-park. This accelerated expansion is driven by a bulk scalar field, whose parameters are constrained by the gauged supergravity theory in the bulk. On top of this, the model predicts a significant evolution of the induced gravitational constant G on the brane. Indeed, the value of G can be seen to have changed by 37 percent since radiation/matter equality. Thus, the study of cosmological perturbation theory in these models is rather important, in particular the time evolution of G may leave traces in the evolution of perturbations. In this paper we discuss the evolution of the density constrast and the effect of the bulk scalar field on cosmological perturbations. In section 2 we review Einstein's equations induced on our brane world and discuss the Friedmann equation. We also review the background solutions found in [10], which are needed in order to solve the perturbation equations. In section 3 we derive the perturbation equations using the fluid flow approach. This approach is very transparent for our purposes and makes it easy to derive the necessary evolution equations. In section 4 we discuss some solutions of these equations and discuss their properties. We point out the differences to the Randall-Sundrum scenario and usual four-dimensional scalar-tensor theories. We conclude in section 5. In the appendix we discuss some details concerning the issue of supersymmetry breaking and conformal flatness.

2. Brane cosmology

In this section we discuss the field equations on the brane and discuss the background evolution.

2.1 The background evolution

We consider our universe to be a boundary of a five dimensional space-time. The embedding is chosen such that our brane-world sits at the origin of the fifth dimension. We impose a $\mathbb{Z}_2$ symmetry along the fifth dimension and identify x_5 with $-x_5$. Our brane-world carries two types of matter, the standard model fields at sufficiently large energy and ordinary matter and radiation at lower energy. We also assume that gravity propagates in the bulk where a scalar field ϕ lives. This scalar field couples to the standard model fields living on the brane-world. At low energy when the standard model fields have condensed and the electro-weak and hadronic phase transitions have taken place, the coupling of the scalar field to the brane-world realizes the mechanism proposed in [12] with a self-tuning of the brane tension. In this section we derive the brane cosmology equations describing the coupling between ordinary matter on the brane and a scalar field in the bulk.

Consider the bulk action

$$S_{\text{bulk}} = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g_5}\left(R - \frac{3}{4}((\partial\phi)^2 + U(\phi))\right), \tag{2.1}$$

where $\kappa_5^2 = 1/M_5^3$ and the boundary action

$$S_B = -\frac{3}{2\kappa_5^2}\int d^4x\sqrt{-g_4}U_B(\phi_0)\,, \tag{2.2}$$

where ϕ_0 is the boundary value of the scalar field. The Einstein equations read

$$G_{ab} \equiv R_{ab} - \frac{1}{2} R g_{ab} = T_{ab} + \delta_{x_5} T^B_{ab}\,, \tag{2.3}$$

where T_{ab} is the bulk energy-momentum tensor and T^B_{ab} is the boundary contribution. The bulk term is

$$T_{ab} = \frac{3}{4}\left(\partial_a\phi\partial_b\phi - \frac{1}{2} g_{ab}(\partial\phi)^2\right) - \frac{3}{8} g_{ab} U \tag{2.4}$$

and the boundary term

$$T^B_{ab} = -\frac{3}{2} g_{ab} U_B(\phi) \tag{2.5}$$

with $a, b = 0 \ldots 3$ in the last equation. Following the self-tuning proposal we interpret U_B as arising from a direct coupling U^0_B to the brane degrees of freedom, i.e. the standard model fields Φ^i. The vacuum energy generated by the Φ^i's yields the effective coupling

$$\frac{3U_B}{2\kappa_5^2} = \langle V(\Phi)\rangle U^0_B\,, \tag{2.6}$$

where the dimension four potential $V(\Phi)$ represents all the contributions due to the fields Φ^i after inclusion of condensations, phase transitions and radiative corrections.

We also consider that ordinary matter lives on the brane with a diagonal energy momentum tensor

$$\tau^a{}_b = \mathrm{diag}(-\rho, p, p, p) \tag{2.7}$$

and an equation of state $p = \omega\rho$. In this paper we will assume that the fluid couples to the bulk scalar field only via gravity.

We will be mainly concerned with models derived from supergravity in singular spaces. They involve $N = 2$ supergravity with vector multiplets. When supergravity in the bulk couples to the boundary in a supersymmetric way the Lagrangian is entirely specified by the superpotential

$$U_B = W \tag{2.8}$$

and the bulk potential

$$U = \left(\frac{\partial W}{\partial\phi}\right)^2 - W^2\,. \tag{2.9}$$

If one considers a single vector supermultiplet then supersymmetry imposes that

$$W = \xi e^{\alpha\phi}\,, \tag{2.10}$$

where $\alpha = 1/\sqrt{3},\ -1/\sqrt{1}2$, these values arising from the parametrisation of the moduli space of the vector multiplets, and ξ is a characteristic scale related to the brane tension.

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Since supersymmetry is not observed in nature, one should incorporate supersymmetry breaking. A natural way to break supersymmetry is by coupling the bulk scalar field to brane fields fixed at their vevs. This leads to

$$U_B = TW\,, \tag{2.11}$$

where $T = 1$ is the supersymmetric case. Larger values of T correspond to supersymmetry breaking effects with a positive energy density on the brane. We will analyse the dynamics of the coupled system comprising gravity, the scalar field ϕ and matter on the brane.

The dynamics of the brane world is specified by the four dimensional Einstein equations [13]

$$\bar{G}_{ab} = -\frac{3V}{8}n_{ab} + \frac{U_B}{4}\tau_{ab} + \pi_{ab} + \frac{1}{2}\nabla_a\phi\nabla_b\phi - \frac{5}{16}(\nabla\phi)^2 n_{ab} - E_{ab}\,, \tag{2.12}$$

where τ_{ab} is the matter energy momentum tensor, n_{ab} the induced metric on the brane and E_{ab} the projected Weyl tensor of the bulk onto the brane. It appears as an effective energy momentum tensor on the brane, called the Weyl fluid [5]. Note that one can identify the four-dimensional Newton's constant with the brane tension [13], i.e.

$$8\pi G_N = \frac{U_B}{4}\,. \tag{2.13}$$

The tensor π_{ab} is quadratic in the matter energy momentum tensor

$$\pi_{ab} = \frac{\tau}{12}\tau_{ab} - \frac{\tau_{ac}\tau_b^c}{4} + \frac{\tau_{ab}}{24}(3\tau_{cd}\tau^{cd} - \tau^2)\,, \tag{2.14}$$

where $\tau = \tau^a{}_a$, The Bianchi identity $\nabla^a G_{ab} = 0$ leads to the conservation equation

$$\nabla^a E_{ab} = \frac{\nabla^a U_B}{4}\tau_{ab} + \nabla^a\pi_{ab} + \nabla^a P_{ab}\,, \tag{2.15}$$

where the tensor P_{ab} is defined by

$$P_{ab} = -\frac{3V}{8}n_{ab} + \frac{1}{2}\nabla_a\phi\nabla_b\phi - \frac{5}{16}(\nabla\phi)^2 n_{ab}\,. \tag{2.16}$$

This consistency equation (2.15) will allow us to follow the time evolution of E_{00} in the background and at the perturbative level. This is crucial in order to define the time evolution of the background and of the scalar field and matter perturbations.

The dynamics of the scalar field is specified by the Klein-Gordon equation which reads

$$\nabla^2\phi + \frac{\tau}{6}\frac{\partial U_B}{\partial\phi} = \frac{\partial V}{\partial\phi} - \Delta\Phi_2\,, \tag{2.17}$$

where we have defined the loss parameter by

$$\Delta\Phi_2 = \partial_n^2\phi|_0 - \frac{\partial U_B}{\partial\phi}\frac{\partial^2 U_B}{\partial\phi^2} \tag{2.18}$$

and $\partial_n^2\phi|_0$ stands for the second normal derivative of the scalar field at the brane location. The effective scalar potential is defined by

$$V = \frac{U + (\partial U_B/\partial\phi)^2 - U_B^2}{2}\,. \tag{2.19}$$

We derive these equations in section 3.

In order to illuminate the role of U_B further, we point out that the projected Klein-Gordon equation can be seen as an equation for the brane energy-momentum tensor

$$\mathcal{T}_{ab} = \nabla_a\phi\nabla_b\phi - \frac{1}{2}((\nabla\phi)^2 + 2V)n_{ab}, \tag{2.20}$$

which reads

$$\nabla^a\mathcal{T}_{ab} = -\frac{\mathcal{T}}{6}\nabla_b U_b - \Delta\Phi_2\nabla_b\phi \tag{2.21}$$

and can be derived from the four dimensional effective action for ϕ

$$S = \int d^4x\sqrt{-g_4}\left((\nabla\phi)^2 + 2V - \frac{\mathcal{T}}{3}U_B - 2\Delta\Phi_2\phi\right), \tag{2.22}$$

whenever $\Delta\Phi_2$ is constant. It is remarkable that U_B plays the role of a dilaton coupled to the trace of the matter energy-momentum tensor.

For the background cosmology the Klein-Gordon equation reduces to (see section 3)

$$\ddot\phi + 4H\dot\phi - \frac{\mathcal{T}}{6}U_B' = -V' + \Delta\Phi_2\,, \tag{2.23}$$

where dot stands for the proper time derivative. The bulk expansion rate is defined by $4H \equiv \partial_\tau \ln\sqrt{-g}|_0$ evaluated on the brane.

The background cosmology is characterized by its isotropy so we consider that $E_{0i} = 0$, $E_{ij} = 0$, $i \neq j$. Moreover we assume that we obtain a FRW induced metric on the brane. Due to the tracelessness of E_{ab} it is then sufficient to obtain the differential equation for E_{00}

$$\dot E_{00} + 4H_B E_{00} = \partial_t\left(\frac{3}{16}\dot\phi^2 + \frac{3}{8}V\right) + \frac{3}{2}H_B\dot\phi^2 + \frac{\dot U_B}{4}\rho\,, \tag{2.24}$$

where H_B is the brane expansion rate $3H_B \equiv \partial_\tau \ln\sqrt{-n}|_0$ as a function of the brane metric n. This leads to

$$E_{00} = \frac{1}{a^4}\int dt a^4\left(\partial_t\left(\frac{3}{16}\dot\phi^2 + \frac{3}{8}V\right) + \frac{3}{2}H_B\dot\phi^2 + \frac{\dot U_B}{4}\rho\right). \tag{2.25}$$

Writing the bulk metric as

$$ds^2 = e^{2A(z,t_b)}(-dt_b^2 + dz^2) + e^{2B(z,t_b)}dx^2\,, \tag{2.26}$$

the proper time on the brane is defined by $dt = e^{A(0,t_b)}dt_b$. We can always choose the boundary condition $A(0,t_b) = B(0,t_b)$ in such a way

$$H = H_B\,. \tag{2.27}$$

Together with the Klein-Gordon equation this leads to

$$E_{00} = \frac{1}{16a^4}\int dta^4\left(\dot{U}_B(4\rho - \tau) + 6\Delta\Phi_2\dot{\phi}\right). \tag{2.28}$$

In particular we find that conformal flatness is broken as soon as ϕ becomes time-dependent and either matter is present on the brane or the energy loss parameter does not vanish. Notice that a sufficient condition for breaking conformal flatness is that Newton's constant becomes time-dependent (see eq. 2.13). We will comment on this expression when discussing the various cosmological eras.

Using

$$\bar{G}_{00} = 3H_B^2 \tag{2.29}$$

and after one integration by parts we obtain the Friedmann equation

$$H_B^2 = \frac{\rho^2}{36} + \frac{U_B}{12}\rho + \frac{1}{16a^4}\int dt\frac{da^4}{dt}(2V - \dot{\phi}^2) - \frac{1}{12a^4}\int dt\frac{dU_B}{dt}\rho + \frac{C}{a^4}\,. \tag{2.30}$$

This equation has already been derived in [10]. We remember that the scalar field ϕ enters in the definition of the effective Newton's constant (see eq. 2.13. The last three terms are a combination of the energy flow onto or away from the brane and the changes of the pressure along the fifth dimension [14].

Using the conservation of matter

$$\nabla_a\tau^a_b = 0 \tag{2.31}$$

we find that

$$\dot{\rho} = -3H_B(\rho + p)\,. \tag{2.32}$$

This completes the description of the three equations determining the brane cosmology, i.e. the Klein-Gordon equation, the Friedmann equation and the conservation equation.

Notice that there are two entities which depend on the bulk. First of all there is the dark radiation term C/a^4 whose origin springs from the possibility of black-hole formation in the bulk. Then there is the loss parameter $\Delta\Phi_2$ which depends on the evolution of the scalar field in the bulk. It specifies the part of the evolution of ϕ, which is not constrained by considerations of the brane dynamics.

In the following we shall describe the case where the bulk theory is $N = 2$ supergravity with vector multiplets. When no matter is present on the brane, the background cosmology can be explicitly solved. In particular the metric is conformally flat implying that

$$C = 0\,. \tag{2.33}$$

Moreover the loss parameter vanishes explicitly

$$\Delta\Phi_2 = 0\,. \tag{2.34}$$

A detailed analysis is presented in the appendix. In particular the last equation implies that the brane dynamics is closed. When matter density is present on the brane, we show in the appendix that conformal flatness is not preserved. Nevertheless we assume that the breaking of conformal flatness is small enough to allow one to assume that (2.33) and (2.34) are still valid, both for the background as well as on the perturbative level.

We now turn to the discussion of the background evolution. The solutions given below have been obtained in [10], but we need to present them in detail in order to discuss the evolution of density perturbations in section 4.

2.2 Radiation dominated eras

The background cosmology can be solved in four different eras: the high energy era, the radiation and matter dominated epochs and finally the supergravity era where the scalar field dynamics dominates. We discuss these four eras in turn paying particular attention to the matter-supergravity transition where we show that requiring coincidence now implies a fine-tuning of the supersymmetry breaking part of the brane tension.

First of all we consider the high energy regime where the non-conventional ρ^2 dominates, i.e. for energies higher than the brane tension. In that case we can neglect the effective potential V and find that

$$a = a_0 \left(\frac{t}{t_0}\right)^{1/4}, \tag{2.35}$$

while the scalar field behaves like

$$\phi = \phi_0 + \beta \ln\left(\frac{t}{t_0}\right). \tag{2.36}$$

In the following we will focus on the case $\beta = 0$, i.e. a constant scalar field, for which the projected Weyl tensor vanishes altogether

$$E_{00} = 0\,. \tag{2.37}$$

Notice that Newton's constant does not vary in time in this case.

The usual radiation era is not modified by the presence of the scalar field

$$\phi = \phi_0 \tag{2.38}$$

and

$$a = a_r \left(\frac{t}{t_r}\right)^{1/2} . \tag{2.39}$$

Newton's constant does not vary in time, while

$$E_{00} = 0 , \tag{2.40}$$

as in the high energy regime. Note that the solution in the radiation era in this brane world scenario is similar to the radiation era solution found in Brans-Dicke theory, see [15] and [16], for example. The field ϕ approaches quickly the attractor for which ϕ =constant.

2.3 Matter dominated era

The matter dominated era leads to a more interesting background cosmology. Let us first consider the pure sugra case where $T = 1$. This is a good approximation until coincidence where the potential energy of the scalar field cannot be neglected anymore. The solution to the evolution equations is

$$\begin{aligned} \phi &= \phi_0 + \beta \ln\left(\frac{t}{t_e}\right) \\ a &= a_e \left(\frac{t}{t_e}\right)^{\gamma} , \end{aligned} \tag{2.41}$$

where τ_e and a_e are the time and scale factors at radiation-matter equality. We are interested in the small α case as it leads to an accelerating universe when no matter is present and small time deviations for Newton's constant.

For small α we get

$$\begin{aligned} \beta &= -\frac{8}{15}\alpha \\ \gamma &= \frac{2}{3} - \frac{8}{45}\alpha^2 . \end{aligned} \tag{2.42}$$

The projected Weyl tensor is given by

$$E_{00} = -\frac{4\alpha^2}{3t^2} , \tag{2.43}$$

which decreases like a^{-3} to leading order in α.

It is useful to compare these results with general relativity, so for the moment we identify in a phenomenological way Newton's constant with the ratio

$$\frac{8\pi G_N(\tau)}{3} \equiv \frac{H^2}{\rho_m} , \tag{2.44}$$

which is now time dependent, whereas in general relativity this ratio is strictly constant. In terms of the red-shift z this is

$$\frac{G_N(z)}{G_N(z_e)} = \left(\frac{z+1}{z_e+1}\right)^{4\alpha^2/5} . \tag{2.45}$$

For the supergravity case with $\alpha^2 = 1/12$ the exponent is $1/15$. As $z_e \sim 10^3$ this leads to a decrease of the quantity (2.44) by 37% since equality.

Notice that Newton's constant, as defined in eq. (2.13), starts to decrease only from the time of matter and radiation equality and is strictly constant during the radiation dominated era. Nucleosynthesis constraints the variation to be less than 20% at the time of nucleosynthesis. In order to confront our theory with local tests for the variation of Newton's constant we need to extract the weak field limit, which is beyond the scope of the paper. In addition, we note that many of the tests for the variation of the Newton constant assume all other masses and couplings are constant [17].

2.4 Broken supergravity era

After coincidence matter does not dominate anymore; this is the supergravity era dominated by the scalar field dynamics. Let us review it briefly. Consider first the pure sugra case. It is easy to see that the potential vanishes

$$V_{\mathrm{SUGRA}} = 0 \tag{2.46}$$

leading to a static universe with

$$\begin{aligned} \phi &= -\frac{1}{\alpha}\ln(1-\alpha^2\xi|y|) \\ a &= (1-\alpha^2\xi|y|)^{1/4\alpha^2} , \end{aligned} \tag{2.47}$$

where we have defined $dy = a dx_5$. This is a flat solution corresponding to a vanishing cosmological constant on the brane-world.

As soon as supergravity is broken on the brane $T \neq 1$ the static solution is not valid anymore. The new four dimensional potential becomes

$$V = \frac{(T^2-1)}{2}\left(W^2 - \left(\frac{\partial W}{\partial \phi}\right)^2\right) . \tag{2.48}$$

The time dependent background is obtained from the static solution by going to conformal coordinates

$$ds^2 = a^2(u)(-d\eta^2 + du^2 + dx^i dx_i) \tag{2.49}$$

and performing a boost along the u axis

$$\begin{aligned} a(u,\eta) &= a\left(u+h\eta, \frac{\xi}{\sqrt{1-h^2}}\right) \\ \phi(u,\eta) &= \phi\left(u+h\eta, \frac{\xi}{\sqrt{1-h^2}}\right), \end{aligned} \tag{2.50}$$

where we have displayed the explicit ξ dependence. Now for

$$h = \pm\frac{\sqrt{T^2-1}}{T} \tag{2.51}$$

we find that the Friedmann equation is solved [10]. Similarly the Klein-Gordon equation is satisfied. Moreover we find that

$$E_{00} = 0\,, \tag{2.52}$$

as the bulk metric is conformally flat.

The resulting universe is characterized by the scale factor in cosmic time

$$a(t) = \frac{1}{\sqrt{T}}\left(1-\frac{t}{t_0}\right)^{1/3+1/6\alpha^2} \tag{2.53}$$

with $t_0 = \frac{2}{3\alpha^2}\frac{1}{hT^{3/2}\xi}$ and $h < 0$. The scale factor corresponds to a solution of the four dimensional FRW equations with an acceleration parameter

$$q_0 = \frac{6\alpha^2}{1+2\alpha^2} - 1 \tag{2.54}$$

and an equation of state

$$\omega_{\mathrm{SUGRA}} = -1 + \frac{4\alpha^2}{1+2\alpha^2} \tag{2.55}$$

which never violates the dominant energy condition. The solution with $\alpha = -1/\sqrt{12}$ is accelerating. In particular we find that

$$q_0 = -\frac{4}{7} \tag{2.56}$$

and for the equation of state

$$\omega_{\mathrm{SUGRA}} = -\frac{5}{7}\,. \tag{2.57}$$

This is within the experimental ball-park.

2.5 The matter-supergravity transition

Let us now investigate the transition between the matter dominated and supergravity eras. If we denote by $H_{\rm SUGRA}$ the Hubble parameter derived in the unbroken supergravity case, i.e. $T = 1$, then the Friedmann equation in the broken supergravity case is

$$H^2 = H^2_{\rm SUGRA} + \frac{V}{8}\,, \tag{2.58}$$

where we have used the fact that ϕ varies slowly compared to α. The evolution coincides with the one obtained from unbroken supergravity as long as the contribution from the potential does not dominate. In the radiation dominated era this requires

$$\frac{T^2-1}{T}\frac{W}{2\kappa_5^2} \ll \frac{2}{3}\frac{\rho_e}{1-\alpha^2}\,, \tag{2.59}$$

where ρ_e is the matter density at equality. This implies that the left-hand side is much smaller that 10^{-39} GeV^4. Let us now denote the supersymmetric brane tension by

$$M_S^4 = \frac{3W}{2\kappa_5^2} \tag{2.60}$$

and the supersymmetry breaking contribution

$$M_{BS}^4 = (T-1)M_S^4\,. \tag{2.61}$$

We find that

$$M_{BS}^4 \ll \frac{\rho_e}{1-\alpha^2}\,. \tag{2.62}$$

Now this is an extreme fine-tuning of the non-supersymmetric contribution to the brane tension.

In the matter dominated era the supergravity Hubble parameter decreases faster than the potential contribution. Coincidence between the matter dominated supergravity contribution $H^2_{\rm SUGRA}$ and the potential energy occurs at z_c

$$M_{BS}^4 = \frac{1}{1-\alpha^2}\left(\frac{z_c+1}{z_e+1}\right)^{3+\alpha\beta/\gamma}\rho_e\,. \tag{2.63}$$

Imposing that coincidence has occurred only recently leads to

$$M_{BS}^4 \approx \rho_c\,, \tag{2.64}$$

where ρ_c is the critical density. This is the usual extreme fine-tuning of the cosmological constant. Indeed it specifies that the energy density received by the brane-world from the non-supersymmetric sources, e.g. radiative corrections and phase transitions, cannot exceed the critical energy density of the universe.

3. Cosmological perturbations using the fluid flow approach

We now turn to the discussion of cosmological perturbations. As we investigate the brane point of view and do not intend to solve the full five-dimensional equations, we will set $\Delta\Phi_2 = 0$ and $\pi^i{}_j = 0$ for $i \neq j$. The off-diagonal components of $\pi_{\mu\nu}$ are formally an additional stress induced by bulk gravitons [7].

There are different effects which will influence the evolution of perturbations:

- The evolution of the gravitational constant in the matter era changes the evolution of the background: In the matter era, the gravitational constant decreases and furthermore the universe is expanding slower (up to order α^2) than in the FRW matter dominated era in general relativity.
- Perturbations in the scalar field are the source of matter fluctuations and vice versa.
- Perturbations in the projected Weyl tensor act as sources for the scalar and matter perturbations and vice versa.

These effects will change the growth of perturbations compared to normal 4D cosmological models or the Randall-Sundrum model. In particular it should be noted that our formalism allows us to treat the Randall-Sundrum cosmology. Indeed by putting $\alpha = 0$, $T = 1$ and neglecting the scalar field contribution we obtain the Randall-Sundrum case with a flat boundary brane while putting $T \neq 1$ leads to a de Sitter boundary brane.

While discussing perturbations, we use the fluid-flow approach (see [18, 19] and [20]) rather than the metric-based approach [6]. The main difference is that all perturbation variables are expressed in terms of fluid-quantities, rather than metric-variables. For our purpose, i.e. discussing the evolution of the density contrast $\delta = \delta\rho/\rho$ of the dominant fluid at each epoch, this approach is simpler in order to obtain the evolution equations. To do so, we need to derive the Raychaudhuri equation and the Klein-Gordon equation in the comoving frame.

3.1 The Raychaudhuri and Klein-Gordon equation

On the brane $\mathcal{B}$, see figure 1, the matter energy-momentum tensor is conserved. Denoting by n the normal vector to the brane and defining the induced metric by

$$n_{ab} = g_{ab} - n_a n_b \tag{3.1}$$

such that $n^2 = 1$ and $n_{ab}n^b = 0$, the conservation equation reads

$$\nabla_a \tau^{ab} = 0\,, \tag{3.2}$$

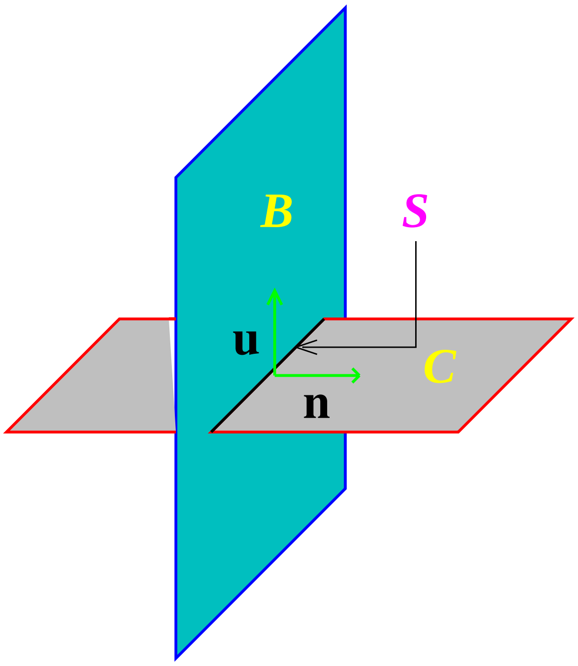

Figure 1: The brane-world $\mathcal{B}$ is perpendicular to the normal vector n. The four dimensional hypersurface $\mathcal{C}$ is orthogonal to the velocity vector u. The surface $S = C \cap B$ is a set of comoving observers following matter on the brane. Note that the perturbed brane is not necessarily located at $y =$constant. The different metrics in the text are as follows: g_{ab} is the full five-dimensional metric, n_{ab} is the induced metric on the brane $\mathcal{B}$, h_{ab} is the induced metric on $\mathcal{C}$ and u_{ab} is the induced metric on the comoving hypersurface $\mathcal{S}$.

where $\nabla_a = n_a^b D_b$ is the brane covariant derivative and

$$\tau_{ab} = (\rho + p) u_a u_b - p n_{ab} \tag{3.3}$$

is the energy-momentum tensor. Notice that the vector u_a is the velocity field of the brane matter and thus must be orthogonal to n and satisfies $u^2 = -1$. In addition we have the usual decomposition in terms of the shear σ_{ab}, the helicity ω_{ab} and the expansion rate

$$\nabla_c u^d = -u_c \dot{u}^d + H_B u_c^d + \sigma_c^d + \omega_c^d \,, \tag{3.4}$$

where we set $\sigma_{ab} = 0,\ \omega_{ab} = 0$ later on.

Using the relations

$$\nabla_a u^a = 3H_B \,, \qquad \dot{u}_a = u^b \nabla_b u_a \tag{3.5}$$

one derives from (3.2)

$$\dot{\rho} = -3H_B(\rho + p) \,. \tag{3.6}$$

Notice that no matter leaks out of the brane. Defining

$$\bar{\nabla}_a \equiv u_{ab} \nabla^b = \nabla_a + u_a u^b D_b \,, \tag{3.7}$$

which is nothing but the spatial covariant derivative on the brane, one obtains

$$\dot{u}_a = -\frac{\bar{\nabla}_a p}{\rho + p}, \tag{3.8}$$

whose divergence $\nabla_a \dot{u}^a$ leads to the Raychaudhuri equation

$$3\dot{H}_B + 3H_B^2 = -R_{00} - \bar{\nabla}_a \left(\frac{\bar{\nabla}_a p}{\rho + p} \right). \tag{3.9}$$

We have neglected σ_{ab} and ω_{ab} here as we will consider linear scalar perturbations only.

In order to have a closed system of differential equations for the perturbations we need the Klein-Gordon equation in the comoving gauge. The metric on a hypersurface $\mathcal{C}$ orthogonal to u, see figure 1, is given by

$$h_{ab} = g_{ab} + u_a u_b, \tag{3.10}$$

where $u^2 = -1$ and $h_{ab}u^b = 0$. Defining the covariant derivative on the hypersurface $\mathcal{C}$ by $\bar{D}_a = h_{ab}D^b$ and using $D_a u^a = 4H$ one obtains

$$D_a D^a = \bar{D}_a \bar{D}^a - 4H u_c D^c - u_a u_b D^a D^b. \tag{3.11}$$

In the comoving gauge we have $u_0 = -1, u_i = 0$ leading to

$$D^2 \phi = \bar{D}^2 \phi - 4H\dot{\phi} - \ddot{\phi}. \tag{3.12}$$

We can now evaluate the laplacian $\bar{D}^2$ in terms of the laplacian on the comoving surface $\mathcal{S}$ (see figure 1) orthogonal to u. Notice that $\bar{\nabla}_a = n_{ab}\bar{D}^b$. Expanding $\bar{\nabla}^2$ leads to

$$\bar{D}^2 = \bar{\nabla}^2 + (K + u^a u^b K_{ab}) n_c \bar{D}^c + n_a n_b \bar{D}^a \bar{D}^b. \tag{3.13}$$

Now $K_{ab}u^a u^b = u^a D_a(u \cdot n) - \dot{u}_a n^a$, where

$$K_{ab} = D_a n_b \tag{3.14}$$

is the extrinsic curvature tensor. Using $(u \cdot n) = 0$ and $n^a \bar{\nabla}_a p = 0$ we find that

$$K_{ab} u^a u^b = 0. \tag{3.15}$$

The junction conditions lead to

$$K = \frac{\tau}{6} - U_B, \tag{3.16}$$

with $\tau = -\rho + 3p$. Finally we can read off the Klein-Gordon equation

$$\ddot{\phi} + 4H\dot{\phi} - \phi'' - \left(\frac{\tau}{6} - U_B \right) \phi' - \bar{\nabla}^2 \phi = -\frac{1}{2} \frac{\partial U}{\partial \phi}, \tag{3.17}$$

where prime stands for the normal derivative. Now the junction conditions lead to

$$\phi' = \frac{\partial U_B}{\partial \phi}\,. \tag{3.18}$$

We find that the Klein-Gordon equation reduces to

$$\ddot{\phi} + 4H\dot{\phi} + \frac{1}{6}\frac{\partial U_B}{\partial \phi}\,(\rho - 3p) - \bar{\nabla}^2\phi = -\frac{\partial V}{\partial \phi} + \Delta\Phi_2\,. \tag{3.19}$$

Notice that it involves only brane derivatives when the loss parameter $\Delta\Phi_2 = 0$. We now turn to the evolution equation for E_{00} which enters in the evaluation of R_{00}.

3.2 The evolution equation for $E_{\mu\nu}$

Let us consider the hypersurface $\mathcal{S}$ on the brane with induced metric u_{ab} and orthogonal to the velocity vector u^a such that $u_{ab}u^a = 0$ and $u^a u_a = -1$. The induced metric is given by $u_{ab} = n_{ab} + u_a u_b$, where n_{ab} is the brane metric. Let ∇_a be the brane covariant derivative and $\bar{\nabla}_a$ the covariant derivative with respect to u_{ab}.

Consider now $\nabla^a E_{ab}$:

$$\nabla^a E_{ab} = u^{ac}\nabla_c E_{ab} - u^a u^c \nabla_c E_{ab}\,. \tag{3.20}$$

The last term can be written as

$$u^a u^c \nabla_c E_{ab} = u^c \nabla_c (u^a E_{ab}) - \dot{u}^a E_{ab}\,. \tag{3.21}$$

The first term in equation (3.20) can be rewritten as

$$\begin{aligned} u^{ac}\nabla_c E_{ab} &= \nabla_c(u^{ac}E_{ab}) - E_{ab}(\nabla_c u^{ac}) \\ &= \nabla_c(u^{ac}E_{ab}) - E_{ab}\,(\dot{u}^a + 3H_B u^a)\,. \end{aligned} \tag{3.22}$$

As a next step we consider

$$\begin{aligned} \nabla_c(u^{ac}E_{ab}) &= \nabla_c(u^{ac}n^d{}_b E_{ad}) \\ &= \nabla_c(u^{ac}u^d{}_b E_{ad}) - \nabla_c(u^{ac}u^d u_b E_{ad}) \\ &= \nabla_c(u^{ac}u^d{}_b E_{ad}) - u^d\nabla_c(u^{ac}u_b E_{ad}) - (\nabla_c u^d)u^{ac}E_{ad}u_b\,. \end{aligned} \tag{3.23}$$

Using (3.4)

$$(\nabla_c u^d)u^{ac}E_{ad}u_b = H_B u^{ad}E_{ad}u_b + (\sigma_c{}^d + \omega_c{}^d)u^{ac}E_{ad}u_b\,. \tag{3.24}$$

Let us further define the projected tensor

$$\bar{E}_{ab} = u_a{}^c u_b{}^d E_{cd}\,. \tag{3.25}$$

Then

$$\nabla_c \bar{E}^c{}_d = \bar{\nabla}_c \bar{E}^c{}_d - \dot{u}_c \bar{E}^c{}_d\,. \tag{3.26}$$

So, from eq. (3.23) we get

$$\nabla_c(u^{ac}E_{ab}) = \bar{\nabla}_c\bar{E}^c{}_b - \dot{u}_c\tilde{E}^c{}_b - u^d\nabla_c(u^{ac}E_{ac}u_b) - H_B u^{ad}E_{ad}u_b - \\ -(\sigma_c{}^d + \omega_c{}^d)u^{ac}E_{ad}u_b\,. \tag{3.27}$$

Collecting everything and defining $\bar{E} = u^{ab}E_{ab}$,the spatial trace, we end up with

$$\nabla^a E_{ab} = \bar{\nabla}_c\bar{E}^c{}_b - \dot{u}_c\bar{E}^c{}_b - u^d\nabla_c(u^{ac}E_{ad}u_b) - (u^aE_{ab})^{\cdot} - H_B\bar{E}u_b - 3H_Bu^aE_{ab} - \\ -(\sigma_c{}^d + \omega_c{}^d)u^{ac}E_{ad}u_b. \tag{3.28}$$

Let us now specialize to the comoving gauge, i.e. the surface $\mathcal{S}$ is a comoving one and $u^0 = 1,\ u^i = 0$. Notice that the first term of the last expression vanishes. We obtain

$$\nabla^a E_{a0} = \bar{\nabla}_c E^c_0 - 4H_BE_{00} - \dot{E}_{00} - \dot{u}_aE^a_0 + (\sigma^d_c + \omega^d_c)u^{ac}E_{ad}\,, \tag{3.29}$$

where we have used the tracelessness of E_{ab} to get $\bar{E} = E_{00}$. For the background we retrieve the previous expression as $\dot{u}_a = 0$ and $E^i_0 = 0$ and we explicitly assume that $\sigma = 0,\ \omega = 0$ for the background.

We need further to evaluate (see eq. (2.14))

$$\nabla^a\pi_{ab} = \bar{\nabla}^c\pi_{c0} - 3H_B\pi_{00} - H_B\bar{\pi} - \dot{u}_a\pi^a_0 - \dot{\pi}_{00} \tag{3.30}$$

which simplifies to

$$\nabla^a\pi_{ab} = \bar{\nabla}^c\pi_{c0} - \dot{u}_a\pi^a_0 \tag{3.31}$$

and finally we need (see eq. (2.16))

$$\nabla^aP_{a0} = \bar{\nabla}^cP_{c0} - 3H_BP_{00} - H_B\bar{P} - \dot{u}_aP^a_0 - \dot{P}_{00}\,. \tag{3.32}$$

Collecting all these ingredients leads to the consistency equation (2.15) expressed in the comoving frame.

3.3 The perturbed dynamics

We have now obtained all the necessary equations in order to derive the perturbation equations up to linear order. In doing so we decompose all fluid quantities into an average (over the comoving hypersurface) of the quantity plus a perturbation, i.e. $\rho(x,t) = \rho_b(t) + \delta\rho(x,t)$, and similar for the pressure and the expansion rate H. We insert these expressions for all quantities into the equations we obtained above and subtract the average.

We begin with the Raychaudhuri equation and energy conservation equation evaluated in a comoving basis. The Raychaudhuri equation reads

$$3\dot{H} + 3H^2 = -\frac{\nabla^2 p}{\rho + p} - R_{00}\,. \tag{3.33}$$

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In this expression H is the expansion rate, p the pressure and ρ the energy density. All quantites are measured with respect to a comoving observer.[1] R_{00} is the time-time component of the Ricci tensor. The dot represents the time-derivative with respect of the comoving observer (i.e. proper time). The Raychaudhuri equation expresses just the behaviour of matter under the influence of geometry and is independent of the field equation. In particular, for matter on the brane, it is the same as in the usual 4D case. The five-dimensional character of the spacetime enters only through R_{00}. For the FRW metric, R_{00} is

$$R_{00} = -\frac{3}{8}V + \frac{U_b}{8}(\rho + 3p) + \frac{1}{12}\rho(2\rho + 3p) - \frac{7}{16}\dot{\phi}^2 - E_{00}\,. \tag{3.34}$$

The proper time is not a unique label of the comoving hypersurfaces, because it can vary in space. Therefore, we need to transform to coordinate time, which labels these comoving hypersurfaces. This transformation from proper time to coordinate time is given by (see [19] or [20])

$$\frac{dt_{pr}}{dt} = 1 - \frac{\delta p}{\rho + p}\,. \tag{3.35}$$

This gives the variation of proper time due to the perturbations. Using the conservation equation we obtain

$$\delta H = H\frac{\omega}{1+\omega}\delta - \frac{\dot{\delta}}{3(1+\omega)}\,. \tag{3.36}$$

where we have defined the density contrast

$$\delta = \frac{\delta\rho}{\rho}\,. \tag{3.37}$$

In the rest of the paper we denote by dot the coordinate time derivative of the perturbations. In this paper we concentrate on the case of $w = p/\rho =$ const. and $c_s^2 = w$. We deduce that

$$\delta\dot{H} = \frac{\omega}{1+\omega}(H\dot{\delta} + \dot{H}\delta) - \frac{\ddot{\delta}}{3(1+\omega)}\,, \tag{3.38}$$

which combined with the perturbed Raychaudhuri equation

$$\delta\dot{H} = \frac{\delta p}{\rho + p}\dot{H} - 2H\delta H + \frac{1}{3}\frac{k^2}{a^2}\frac{\delta p}{\rho + p} - \frac{1}{3}\delta R_{00} \tag{3.39}$$

leads to an equation for the density constrast

$$\ddot{\delta} + (2 - 3\omega)H\dot{\delta} - 6\omega(H^2 + \dot{H})\delta = (1+\omega)\delta R_{00} - \omega\frac{k^2}{a^2}\delta\,, \tag{3.40}$$

[1] This observer is, of course, confined onto the brane world.

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where we have used $\bar{\nabla}^2 = k^2/a^2$. Notice that the left-hand side coincide with the usual four dimensional expression. The new physical ingredients all spring from δR_{00}. As the Friedmann equation and its time-derivative are given by complicated expressions, we do not derive a general equation for the evolution of δ but rather analyse each regime separately and derive the corresponding equation.

In the same fashion, we derive the perturbed Klein-Gordon equation, using eq. (3.35):

$$\begin{aligned}(\delta\phi)^{\cdot\cdot\cdot} + 4H(\delta\phi)^{\cdot} + \left[\frac{k^2}{a^2} + \frac{\rho}{6}(1-3w)\frac{\partial^2 U_B}{\partial\phi^2} + \frac{\partial^2 V}{\partial\phi^2}\right]\delta\phi = \\ = \frac{c_s^2}{1+\omega}\left[\dot{\phi}\dot{\delta} + \delta(\ddot{\phi} + 4H\dot{\phi})\right] + \frac{1}{6}\frac{\partial U_B}{\partial\phi}\left[3\delta p - \delta\rho\right].\end{aligned} \tag{3.41}$$

where the coupling between the matter and scalar perturbations is explicit.

Finally we need the perturbed version of the E_{00} consistency equation. From the equations we derived in the previous section we obtain

$$\delta(\nabla^a E_{a0}) = \bar{\nabla}^c \delta E_{c0} - 4\delta H_B \dot{E}_{00} - 4H_B \delta E_{00} - \delta\dot{E}_{00} - \frac{\delta p}{p+\rho}\dot{E}_{00}\,, \tag{3.42}$$

where the shear term vanishes automatically at first order. Similarly we have

$$\nabla^a \delta\pi_{a0} = \bar{\nabla}^c \delta\pi_{c0} - \dot{u}_a \pi^a_0 \tag{3.43}$$

and

$$\nabla^a \delta P_{a0} = \bar{\nabla}^c \delta P_{c0} - 3\delta H_B P_{00} - 3H_B \delta P_{00} - \delta H_B \bar{P} - H_B \delta\bar{P} - \dot{u}_a P^a_0 - (\delta P_{00})^{\cdot}\,. \tag{3.44}$$

This allows us to write down the necessary perturbed equation.

Notice that the components of E_{a0} play a role in the perturbed dynamics. As can be seen, they are not constrained by the brane dynamics and lead to a direct influence of the bulk perturbations on the brane perturbations. For this reason the perturbed dynamics is not closed on the brane. In the rest of this paper we will only deal with long wave-length phenomena where the dynamics is closed.

4. Time evolution of cosmological perturbations

In this section we discuss some solutions to the perturbed dynamics. The equations are very difficult to solve in general so we restrict ourselves to the different cosmological eras. We start in the high-energy regime, assuming the universe to be dominated by relativistic particles. Then we discuss the normal (low-energy) radiation dominated epoch, followed by the matter dominated epoch. Because the system is not closed on the brane for large k (small wavelength), we present solutions only for the $k \to 0$ limit. This limit enables us to deduce the effects of the bulk scalar field and $E_{\mu\nu}$ on density perturbations.

In particular we can write the perturbed $E_{\mu\nu}$ equation in that limit as

$$\begin{aligned}
&\delta\dot{E}_{00} + \frac{\delta p}{\rho+p}\dot{E}_{00} + 4H_B\delta E_{00} + 4\delta H_B E_{00} = \\
&\quad = -\frac{\delta\dot{U}_B}{4}\rho - \frac{\delta p}{\rho+p}\frac{\dot{U}_B\rho}{4} - \frac{\dot{U}_B\rho}{4}\delta + \delta\dot{P}_{00} + \frac{\delta P}{\rho+p}\dot{P}_{00} + 3H_B\delta P_{00} + 3\delta H_B P_{00} + \\
&\qquad + H_B\delta\bar{P} + \delta H_B\bar{P}\,.
\end{aligned} \tag{4.1}$$

Explicitly we need

$$P_{00} = \frac{3V}{8} + \frac{3\dot{\phi}^2}{16},\ \bar{P} = -\frac{9V}{16} + \frac{15\dot{\phi}^2}{16}\,. \tag{4.2}$$

The perturbed quantities are then

$$\delta\dot{U}_B = U_B'\delta\dot{\phi} \tag{4.3}$$

and

$$\delta P_{00} = \frac{3}{8}\dot{\phi}\delta\dot{\phi} \tag{4.4}$$

as we neglect the potential in the different eras. This equation will be made more explicit in each era.

As a first step we will first analyse cosmological versions of the Randall-Sundum scenario with a single boundary brane, i.e. we consider a brane embedded in Anti-de Sitter bulk spacetime. The differential equation for the density contrast is of third order leading to the appearance of three modes. This is to be compared to the usual two FRW cosmology modes derived from four dimensional general relativity. As already mentioned two of the modes will coincide with FRW cosmology while the third mode is entirely due to perturbations in the Weyl tensor signalling a breaking of conformal invariance in the bulk.

4.1 The Randall-Sundrum scenario

4.1.1 The high energy era

In the high energy regime, the equations for δE_{00} and δ read

$$\begin{aligned}
\ddot{\delta} + H\dot{\delta} - 18H^2\delta &= -\frac{4}{3}\delta E_{00}\,, \\
(\delta E_{00})^{\cdot} + 4H(\delta E_{00}) &= 0\,.
\end{aligned} \tag{4.5}$$

This leads to

$$\delta E_{00} = \frac{\delta E_0}{t}\,. \tag{4.6}$$

The overall solution of the equation for the density contrast is a sum of the solution of the homogeneous equation as well as the general solution. It can easily be found as

$$\delta = \delta_0 t^{3/2} + \delta_1 t^{-3/4} + \frac{16}{51}\delta E_0 t\,. \tag{4.7}$$

4.1.2 The low-energy radiation era

In this era, the equations for δE_{00} and δ read

$$\begin{aligned} \ddot{\delta} + H\dot{\delta} - 2H^2\delta &= -\frac{4}{3}\delta E_{00}\,, \\ (\delta E_{00})^{\cdot} + 4H(\delta E_{00}) &= 0\,. \end{aligned} \tag{4.8}$$

Again, the solution to the last equation is

$$\delta E_{00} = \frac{\delta E_0}{t^2}\,. \tag{4.9}$$

The full solution to the first equation is found to be

$$\delta = \delta_0 t + \delta_1 t^{-1/2} + \frac{8}{3}\delta E_0\,. \tag{4.10}$$

We do, therefore, find the normal growing and decaying modes in this regime and a constant mode, which is absent in FRW cosmology.

4.1.3 The matter dominated epoch

Finally, in the matter dominated epoch the equation for δE_{00} and δ read

$$\begin{aligned} \ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}H^2\delta &= -\delta E_{00}\,, \\ (\delta E_{00})^{\cdot} + 4H(\delta E_{00}) &= 0\,. \end{aligned} \tag{4.11}$$

The solution to the last equation is

$$\delta E_{00} = \delta E_0 t^{-8/3}\,, \tag{4.12}$$

and the solution to the first equation can then found to be

$$\delta = \delta t^{2/3} + \delta_1 t^{-1} + \frac{9}{4}\delta E_0 t^{-2/3}\,. \tag{4.13}$$

We recover thus the normal growing and decaying mode as well as a mode sourced by δE_{00}, which is decaying rapidly.

We now turn to the case with a scalar field in the bulk. The previous Randall-Sundrum modes will be modified in two ways. First of all the brane potential will lead to slight deviations of the mode exponents similar to the modification of the scale factor exponent in the matter era. Then there will also appear new modes due to the fluctuations of the scalar field governed by the Klein-Gordon equation. We will now analyse each of the different regimes in turn.

4.2 Effects of the bulk scalar field on cosmological perturbations

4.2.1 The high-energy regime

In this regime, the terms which are quadratic in ρ and p dominate both in the background and in the perturbation equations. We assume, that relativistic particles dominate the expansion, i.e. $p = \rho/3$.

Considering the large wavelength limit is sufficient because all cosmologically relevant scales are far outside the horizon. The perturbation equations in this regime are

$$\begin{aligned} \ddot{\delta} + H\dot{\delta} &= 18H^2\delta - \frac{4}{3}\delta E_{00}\,, \\ (\delta\phi)^{\cdot\cdot} + 4H(\delta\phi)^{\cdot} &= 0\,, \\ (\delta E_{00})^{\cdot} + 4H\delta E_{00} &= -\frac{\alpha U_B \rho}{4}(\delta\phi)^{\cdot}\,, \end{aligned} \tag{4.14}$$

where we have assumed that $3H(\delta\phi)^{\cdot} \gg V\delta\phi$.

To obtain the solutions to these equations, we first consider the Klein-Gordon equation. The solutions are

$$\delta\phi = \delta\phi_0 + \delta\phi_1 \ln t\,, \tag{4.15}$$

where $\delta\phi_0$ is a constant mode. We can now find δE_{00}.

$$\delta E_{00} = \frac{\delta E_0}{t} - \frac{3\alpha U_B}{8}\delta\phi_1 \frac{\ln t}{t}\,. \tag{4.16}$$

Notice that the solution comprises two parts. The $1/t$ mode is a solution of the homogeneous equation while the logarithmic mode solves the complete equation. In the following we will always find that the solutions are expressible as a sum of homogeneous modes and modes solving the complete differential equations.

We can now deduce the density constrast

$$\delta = \delta_0 t^{3/2} + \delta_1 t^{-3/4} + \delta_2(t)\,, \tag{4.17}$$

where the complete solution reads

$$\delta_2(t) = -\frac{4}{3}t^{3/2}\int^t dt'(t')^{-13/4}\int^{t'} dt''\delta E_{00}(t'')^{7/4}\,. \tag{4.18}$$

In the long time regime, we focus on the leading growing mode, obtained by approximating

$$\delta_2(t) = O((\ln t)t)\,. \tag{4.19}$$

This implies that we find two leading growing modes in $t^{3/2}$ and $(\ln t)t$. Notice that the logarithmic mode is triggered by the scalar fluctuation, $\delta\phi_1$, and is therefore absent in the Randall-Sundrum case. Moreover for very long times we see that the leading mode increases like $t^{3/2}$, which is much larger than in FRW cosmology. Interestingly, this growth of fluctuations is anomalous, in the sense that the exponent of the growing mode is larger than one. The reason for this is the source term $(18H^2\delta)$ which is much larger than the normal $4\pi G\rho_m\delta$. In addition, there is the contribution of bulk gravity. In this regime we cannot expect the normal behaviour, because gravity is simply not four-dimensional.

4.2.2 Radiation domination

In this regime the Newton's constant is not varying in time. Moreover matter does not appear in the Klein-Gordon equation. This leads to the perturbation equations for the scalar field and the density contrast

$$\begin{aligned}
(\delta\phi)^{\cdot\cdot} + 4H(\delta\phi)^{\cdot} &= 0\,,\\
\ddot{\delta} + H\dot{\delta} - 2H^2\delta &= 4\alpha H^2\delta\phi - \frac{4}{3}\delta E_{00}\,,\\
(\delta E_{00})^{\cdot} + 4H\delta E_{00} &= -3\alpha H^2(\delta\phi)^{\cdot}\,.
\end{aligned} \tag{4.20}$$

As before we find homogeneous and complete solutions. More specifically

$$\delta\phi = \delta\phi_0 + \delta\phi_- t^{-1}\,, \tag{4.21}$$

leading to the perturbed δE_{00}

$$\delta E_{00} = \frac{\delta E_0}{t^2} + \frac{3\alpha}{4}\delta\phi_-\frac{\ln t}{t^2}\,. \tag{4.22}$$

Notice that the first mode springs from the homogeneous equation and the last one from the complete equation. The density contrast can then be deduced:

$$\delta = \delta_1 t + \delta_{-1/2}t^{-1/2} + t\int^t dt'(t')^{-5/2}\int^{t'}\left(\frac{\alpha}{(t'')^{1/2}}\delta\phi - \frac{4}{3}(t'')^{3/2}\delta E_{00}\right). \tag{4.23}$$

There are two homogeneous modes as in FRW cosmology. New contributions emerge from the scalar field. In particular there is a growing mode in $O(\delta\phi_-\ln t)$ which is triggered by the decreasing scalar mode in $\delta\phi_-/t$.

4.2.3 Matter domination

In this regime Newton's constant varies in time as the background scalar field is time dependent. This leads to a very rich structure of modes for the density contrast. The perturbation equations read

$$\begin{aligned}
\ddot{\delta} + 2H\dot{\delta} &= \frac{3}{2}\alpha H^2\left[\delta\phi + \frac{1}{\alpha}\delta\right] - \delta E_{00}\,,\\
(\delta\phi)^{\cdot\cdot} + 4H(\delta\phi)^{\cdot} + 2\alpha^2H^2(\delta\phi) &= -2\alpha H^2\delta\,,
\end{aligned}$$

$$
\begin{aligned}
(\delta E_{00})^{\cdot} + 4H(\delta E_{00}) + 4(\delta H)E_{00} = & -3\alpha H^2(\delta\phi)^{\cdot} - 3\alpha H^2\frac{\beta}{t}\delta + \\
& + \frac{3\beta}{8}\left[-t^{-2}(\delta\phi)^{\cdot} + t^{-1}(\delta\phi)^{\cdot\cdot}\right] + 3H\frac{\beta}{t}(\delta\phi)^{\cdot} - \\
& - \frac{1}{2}\beta^2 t^{-2}\dot{\delta}\,. \qquad (4.24)
\end{aligned}
$$

This system of equations possesses power law solutions, which we derive up to order α^2. Defining each mode by the ansatz

$$
\delta = \delta_i t^{a_i}\,, \qquad \delta\phi = \delta\phi_i t^{b_i}\,, \qquad \delta E_{00} = \delta E^i_{00} t^{c_i} \qquad (4.25)
$$

we find that

$$
a_i = b_i = c_i + 2 \qquad (4.26)
$$

for all modes. There are two types of modes. Let us first discuss the complete solutions of the coupled differential equations. We find that there are three modes corresponding to the exponents

$$
a_{2/3} = \frac{2}{3} - \frac{11624}{7875}\alpha^2\,, \quad a_{-1} = -1 - \frac{1036}{375}\alpha^2\,, \quad a_{-2/3} = -\frac{2}{3} + \frac{808}{225}\alpha^2\,. \qquad (4.27)
$$

The modes $\delta_{2/3}$ and δ_{-1} are deformations of the FRW modes due to the scalar field. In particular we find that $a_{2/3} \approx 0.55$. Thus, the growth of fluctuations is smaller, than in the normal matter dominated epoch. The perturbation of the bulk scalar field is growing with the same exponent as the density constrast and δE_{00} is rapidly decaying with an exponent $c = a - 2 \approx -1.45$.

In addition to the growing mode, there are two decaying modes. One of these decaying modes is a modification of the usual $a = -1$ decaying mode. We obtain $a = -1.23$. Thus, the mode is decaying even faster than normal. The other decaying mode corresponds to $a = -0.37$.

On top of these three modes there are seven homogeneous modes corresponding to zero modes of the various differential operators appearing in the three coupled equations. Let us discuss them in some detail as they have remarkable properties. The homogeneous modes of the perturbed Raychaudhuri equation have characteristic exponents

$$
\tilde{a}_{2/3} = \frac{2}{3} + \frac{16}{225}\alpha^2\,, \qquad \tilde{a}_{-1} = -1 + \frac{64}{225}\alpha^2\,. \qquad (4.28)
$$

Notice that these modes are two deformations of the usual FRW modes. The scalar field potential lifts the degeneracy between the doublets of $a = 2/3$ and $a = -1$ modes. We find that $\tilde{a}_{2/3} \approx 0.67$ very close to the FRW value. That the growing mode is larger than the one found in general relativity comes from the fact that the matter dominated universe is expanding *slower* in the brane world model. Therefore, overdensities, which want to contract but have to compete with the cosmological expansion, can grow more easily in the brane world model.

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The homogeneous modes of the Klein-Gordon equation are characterized by the exponents

$$a_0 = -\frac{8}{15}\alpha^2\,, \qquad a_{-5/3} = -\frac{5}{3} + \frac{56}{45}\alpha^2\,, \tag{4.29}$$

corresponding to an almost constant mode and a mode which decreased very fast.

The E_{00} equations comprises three differential operators. The first order differential operator acting on δE_{00} leads to a single mode with

$$a_{-2/3} = -\frac{2}{3} + \frac{32}{45}\alpha^2\,. \tag{4.30}$$

The differential operator acting on $\delta\phi$ leads to two modes

$$\tilde{a}_0 = 0\,, \qquad a_{-10} = -10 + \frac{512}{135}\alpha^2\,. \tag{4.31}$$

Eventually the operator acting on δ leads to

$$\bar{a}_0 = \frac{10}{27}\,. \tag{4.32}$$

Notice that in the epoch of accelerated expansion the structures get frozen in. We have thus described the perturbation modes in the four cosmological eras.

In conclusion, we have found two the growing modes, with exponent $\tilde{a}_{2/3} \approx 0.67$ and $a_{2/3} \approx 0.55$. The 00-component of the Weyl-tensor is always decaying, whereas perturbations in ϕ follow the density contrast. It is instructive to compare these results to the ones found in Brans-Dicke theory. Here the exponent of the growing and decaying modes are modified as follows [21]:

$$a_+ = \frac{2}{3} + \frac{2}{3\omega} \qquad \text{and} \qquad a_- = -1 - \frac{1}{3\omega}\,, \tag{4.33}$$

where ω is the Brans-Dicke parameter. Note, that there are more modes in these theories as well. Here we find that the growing and decaying modes are shifted due to the presence of a scalar field which couples to gravity in a way which differs from Brans-Dicke theory.

4.2.4 Observational consequences

Although we have focused on the large wavelength limit only, we can draw some (qualitative) conclusions from our findings. First of all, which of the two growing modes in the matter dominated epoch appear, depends on the initial conditions imposed in the early universe. It is likely that both modes are generated. If the mode with exponent $a \approx 0.67$ is not generated and the power spectrum is normalized to the observations today (such as to galaxy clusters), the brane world model would have much more power in the matter perturbations than one would expect in normal quintessence models, for example. Thus, in this case there should be more galaxy clusters at high redshift in the brane world model than in normal Einstein gravity with comparable matter density (with a cosmological constant or quintessence field). If the mode with exponent $a \approx 0.67$ contributes significantly to perturbations, then the power in the perturbations in both theories would be similar. However, much more work is needed in order to make this more concrete.

Another consequence is a modification of the spectrum of anisotropies in the cosmic microwave background radiation (CMB). This is because the distance to the last scattering surface (LSS) will be modified due to the slower expansion in the matter dominated epoch. As a result, the first peak will be shifted to larger angular scales. However, we believe this effect to be small to be detected. In addition to this effect, it is conceivable that isocurvature modes (between the Weyl fluid and radiation, for example) might survive quite long in the radiation era, leaving their imprint in the CMB. To make more concrete predictions, it is necessary to go beyond the small k-limit, which we considered in this paper. This involves a study of the bulk gravitational field, as well as perturbations in the bulk scalar field away from the brane [7]. The calculations are very difficult and a detailed discussion of the bulk equations is beyond the scope of this paper and will be presented elsewhere.

5. Conclusions

We have presented a discussion of cosmological perturbations in brane world scenarios with a bulk scalar field. For a model motivated from SUGRA in singular spaces, we were able to find solutions of the perturbation equations in the large scale limit. As is the case with brane world scenarios of the Randall-Sundrum type, the pertrubation equations on the brane are not closed. Instead one has also to solve the bulk equations, which is a very difficult task in general. In [7], cosmological perturbations where analysed from the full five dimensional point of view using either the metric based approach or the covariant approach. Here, in this paper we have used another formalism, namely the fluid flow approach introduced by Hawking [18]. For our purposes this formalism allowed us to derive easily the evolution equation for the density contrast.

Our findings indicate that there are considerable differences to the Randall-Sundrum scenario as well as to usual scalar-tensor theories in four dimensions. The growth of structures is different than in these models. A more elaborate investigation of perturbations, in particular of the bulk perturbations, would be necessary in order to make definitive predictions concerning cluster abundance, large scale structure and the anisotropies in the CMB.

There are other open questions, which our work leaves. As discussed in [22], in models based on SUGRA in singular spaces we expect a second brane in the bulk at some distance from our brane universe. We have not addressed the influence of this mirror brane on the structures on our brane universe. Similarly, we haven't

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discussed the dynamics of the radion in these models. The effects of these should be encompassed in the Weyl tensor. We will, however, turn to these questions in future work.

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A. Supersymmetric backgrounds

A.1 BPS configurations

In this appendix we will consider configurations which preserve supersymmetry in the bulk. To do so we will look for Killing spinors satisfying the identities

$$\delta_\epsilon \psi_{\mu i} = 0\,, \qquad \delta_\epsilon \lambda_i^x = 0\,, \tag{A.1}$$

where $\psi_{\mu i}$ is the gravitino spinor ($i = 1, 2$) and λ_i^x belongs to the vector multiplet comprising the scalar field ϕ^x. This leads to the first order equations

$$D_\mu \epsilon_i + \frac{i}{8}\gamma_\mu W Q_{ij}\epsilon^j = 0 \tag{A.2}$$

and

$$i\gamma^\mu \partial_\mu \phi^x \epsilon_i + W^{,x} Q_{ij}\epsilon^j = 0\,, \tag{A.3}$$

where $Q_i^j = Q_a(\sigma^a)_i^j$ and $Q_a Q^a = 1$. Define the vector p_a such that

$$\partial_a \phi^x = p_a \partial_z \phi^x \tag{A.4}$$

and the matrix

$$\Gamma = \frac{p_a \gamma^a}{\sqrt{p^2}} \tag{A.5}$$

such that $\Gamma^2 = 1$. Rotation invariance implies that the only non-zero components are $p_z = 1$ and p_t. The unknown component p_t is fixed by the boundary condition. Now the spinors ϵ_i can be split into positive and negative chiralities

$$\epsilon_i = \epsilon_i^+ + \epsilon_i^- \tag{A.6}$$

satisfying

$$\epsilon_i^\pm = \pm i\Gamma Q_{ij}\epsilon^{\pm j}\,. \tag{A.7}$$

At each point this select the chirality defined by Γ, i.e. we preserve one half of the supersymmetries. Now (A.3) is satisfied provided

$$\partial_z \phi^x = \pm \frac{1}{\sqrt{p^2}} W^{,x} \,. \tag{A.8}$$

This is the BPS equation for supersymmetric configurations. Now choosing the positive sign, we find that the boundary condition at $z = 0$ is automatically satisfied provided

$$T\sqrt{p^2} = 1 \,. \tag{A.9}$$

This leads to the vanishing

$$\Delta \Phi_2 = 0 \,. \tag{A.10}$$

Let us come to the bulk evolution of the Killing spinors. They satisfy

$$D_\mu \epsilon_i + \frac{i}{8}\gamma_\mu W Q_{ij}\epsilon^j = 0 \,. \tag{A.11}$$

This is a first order differential equation which can be solved provided the integrability condition

$$[D_a, D_b]\epsilon_i^+ = R_{abcd}\frac{\gamma^{cd}}{8}\epsilon_i^+ \tag{A.12}$$

is fulfilled. Now one gets

$$[D_a, D_b]\epsilon_i^+ = -\frac{1}{64}W^2\gamma_{ab} + \frac{1}{8}\frac{p_a\gamma_b - p_b\gamma_a}{\sqrt{p^2}}W^{,x}W_{,x}\Gamma)\epsilon_i^+ \tag{A.13}$$

from which we deduce that

$$R_{abcd} = -\frac{1}{16}W^2(g_{ac}g_{bd} - g_{ad}g_{bc}) + \frac{1}{4p^2}W^{,x}W_{,x}(p_a g_{b[d}p_{c]} - p_b g_{a[d}p_{c]}) \,. \tag{A.14}$$

For $W_{,x} = 0$, i.e. at the critical points of the superpotential one recognizes the Riemann tensor of AdS_5. More generally we find that the background geometry which preserves supersymmetry in the bulk is such that the Weyl tensor vanishes

$$W_{abcd} = 0 \tag{A.15}$$

as the Weyl tensors form, in the set of curvature tensors, the complement to the antisymmetrized product of the metric tensor with itself or with a symmetric tensor. This implies that the bulk geometry is conformally flat and can be written as

$$ds^2 = e^{2A(z,t)}(-dt^2 + dz^2 + dx_i dx^i) \,. \tag{A.16}$$

This leads to the vanishing of the projected Weyl tensor

$$E_{ab} = 0 \tag{A.17}$$

in the bulk. Moreover the bulk and brane expansion rates coincide

$$H_B = H\,. \tag{A.18}$$

Notice that (A.14) leads to the Ricci tensor

$$R_{ab} = -\frac{W^2}{4}g_{ab} + \frac{W^{,x}W_{,x}}{4p^2}(p^2 g_{ab} + 3p_a p_b) \tag{A.19}$$

and the curvature scalar

$$a^2 R = -\frac{5W^2}{4} + 2W^{\prime x}W_{,x}\,. \tag{A.20}$$

This implies that Einstein equations

$$R_{ab} - \frac{R}{2}g_{ab} = T_{ab} \tag{A.21}$$

are satisfied with

$$\partial_z A = -\frac{U_B}{4}\,. \tag{A.22}$$

Notice that (A.22) and (A.8) are the BPS equations which lead to the accelerating universe. Solutions of these equations satisfy the Einstein equations and the Klein-Gordon equation.

Let us now consider the second brane where supersymmetry is not broken. Now the Killing spinors for the supersymmetric $T = 1$ case satisfy the projection equation

$$\epsilon_i^{\pm} = \pm i\gamma^5 Q_{ij}\epsilon^{\pm j} \tag{A.23}$$

as $p_t = 0$. This is not compatible with (A.7) as $\Gamma \neq \gamma^5$. The only solution is therefore $\epsilon = 0$, i.e. supersymmetry is completely broken by the non-supersymmetric brane with $T \neq 1$. Notice that the breaking of supersymmetry is global due to the presence of two boundaries respecting incompatible supersymmetries.

So we have shown that breaking supersymmetry on the brane leads to broken $N = 0$ background configurations which still satisfy a system of two first order BPS conditions.

A.2 Breaking conformal flatness

Let us now consider the case where matter and radiation are present on the brane. We will show that one cannot deform the bulk geometry in such a way that the boundary conditions are satisfied. This implies that matter on the brane breaks the conformal flatness of the bulk.

Let us perform a small change of coordinates in the bulk

$$\tilde{x} = x - \xi \tag{A.24}$$

inducing a variation of the bulk metric is

$$\delta g_{ab} = \partial_a \xi_b + \partial_b \xi_a \tag{A.25}$$

and of the scalar field

$$\delta(\partial_a \phi) = \partial_c \phi \partial_a \xi^c, \ \delta(\partial_a \partial_b \phi) = \partial_c \partial_a \phi \partial_b \xi^c + \partial_c \partial_b \phi \partial_a \xi^c \,. \tag{A.26}$$

We also need to evaluate the variation

$$\delta(\partial_z g_{ab}) = \partial_z \xi^c \partial_c g_{ab} + \partial_a \xi^c \partial_z g_{cb} + \partial_b \xi^c \partial_z g_{ca} \,. \tag{A.27}$$

The boundary equation $\delta(\partial_z \phi)|_0 = 0$ leads to

$$\partial_z \xi^z|_0 + p_t \partial_z \xi^t = 0 \,. \tag{A.28}$$

We then find that

$$\delta(\partial_z g_{aa})|_0 = 2\partial_a \xi^a \partial_z g_{aa}|_0 \tag{A.29}$$

with no summation involved. The metric boundary condition at the origin are modified according to

$$\delta\left(\frac{\partial_z g_{aa}}{g_{aa}}\right)|_0 = \frac{\delta(\partial_z g_{aa})}{g_{aa}}|_0 - \frac{\partial_z g_{aa}}{g_{aa}}|_0 \frac{\delta g_{aa}}{g_{aa}}|_0 \tag{A.30}$$

implying that

$$\delta\left(\frac{\partial_z g_{aa}}{g_{aa}}\right)|_0 = -\frac{U_B}{2} \partial_a g^{aa} \xi_a|_0 \tag{A.31}$$

with no summation involved. The g_{ii} boundary condition then reads

$$\frac{U_B}{2} \partial_i g^{ii} \xi_i|_0 = \frac{\rho}{6} \,. \tag{A.32}$$

As the background is x^i independent this cannot be satisfied unless $\rho = 0$. This proves that the bulk metric cannot be smootly obtained from the matterless case by performing a change of coordinates in the bulk. In particular conformal flatness is broken leading to an explicit breaking of supergravity by matter on the brane.

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